Physics U6 Circular Motion Worksheet #3

Orbital Velocity – Orbital Period

Name: ______________________

Solutions

When we have circular motion in space around Earth or similar body, the centripetal force is the force of gravity.

This means that . . .

$$Fc = Fg$$

$$ma_c = GMM/R^2$$

$$m\,(v^2/R) = GMM/R^2$$

$$(v^2/R) = GM/R^2$$

$$v^2/R = GM/R^2$$

$$v^2 = GM/R$$

$$v = \sqrt{GM/R}$$

orbital velocity = v = $\sqrt{GM/R}$

1. Calculate the orbital velocity for a satellite that is 300 km above the surface of the Earth. Calculate R (in meters) carefully.

$R = 300{,}000\,m + 6.38 \times 10^6$

$R = 6.68 \times 10^6\,m$

$$V = \sqrt{\frac{\overbrace{(6.67\times10^{-11})}^{G}\overbrace{(5.98\times10^{24})}^{me}}{\underbrace{6.68\times10^{6}}_{R}}} = 7730\,m/s.$$

2. Calculate the orbital velocity of the moon around the Earth. — $\boxed{\text{Solutions}}$ —

→ use mass of what you are going around, M_{earth}

$$V_{orbit} = \sqrt{\frac{GM}{R}} = \sqrt{\frac{(6.67\times10^{-11})(5.98\times10^{24})}{3.84\times10^{8}}}$$

$$\boxed{V_{ORBIT} = 1019\ m/s}$$

3. Calculate the orbital velocity of the Earth around the Sun. → use mass of what you are going around, M_{sun}.

$$V_{ORBIT} = \sqrt{\frac{GM}{R}} = \sqrt{\frac{(6.67\times10^{-11})(1.98\times10^{30})}{1.5\times10^{11}}}$$

$$\boxed{V_{ORBIT} = 29{,}672\ m/s.}$$

4. Calculate the orbital velocity of a satellite 150 km above the surface of the Moon.

$$V_{ORBIT} = \sqrt{\frac{GM}{R}} = \sqrt{\frac{(6.67\times10^{-11})(7.35\times10^{22})}{1.89\times10^{6}}} = \boxed{1610\ m/s}$$

$$\boxed{\begin{aligned} R &= 1.74\times10^{6} + 150{,}000 \\ &= 1.89\times10^{6} \end{aligned}}$$

Similar to above, starting with

$$Fc = Fg$$

$$ma_c = GMM/R^2$$

$$m\,(4\pi^2R/T^2) = GMM/R^2$$

$$4\pi^2R/T^2 = GM/R^2$$

$$4\pi^2R^3/T^2 = GM$$

$$4\pi^2R^3/GM = T^2$$

we can solve for orbital period T $\quad T = \sqrt{4\pi^2R^3/GM}$

5. Find the period of revolution for a satellite that is 400 km above the surface of the Earth.

$R = 400\,000\,m + 6.38 \times 10^6 = 6{,}780{,}000$

$$T = \sqrt{\frac{4\pi^2R^3}{Gm}} = \sqrt{\frac{(4\pi^2)(6{,}780{,}000)^3}{(6.67\times10^{-11})(5.98\times10^{24})}} = \boxed{5554\text{ sec.}}$$

$= 1.5$ hours

6. Find the period of revolution for the moon around the Earth. use mass of Earth, not moon.

$$T = \sqrt{\frac{4\pi^2R^3}{Gm}} = \sqrt{\frac{(4\pi^2)(3.84\times10^8)^3}{(6.67\times10^{-11})(5.98\times10^{24})}}$$

$\boxed{T = 2{,}367{,}353\text{ sec.}}$ $= 657.6$ hours

$= 27.4$ days.

matches data sheet.

7. a) What altitude above the Earth do you need to put a satellite so that it has a period of 24 hours.

$T = 86,400 \text{ sec}$

$$T^2 = \frac{4\pi^2 R^3}{Gm} \rightarrow \sqrt[3]{\frac{T^2 Gm}{4\pi^2}} = \sqrt[3]{R^3} = R.$$

$$R = \sqrt[3]{\frac{(86,400)^2(6.67\times10^{-11})(5.98\times10^{24})}{4\pi^2}} = \boxed{\underbrace{42250474 \text{ m}}_{4.22\times10^7} = R_{TOTAL}}$$

$-(R_{EARTH} = 6.38\times10^6)$

$= 3.58\times10^7 \text{ m}$

(Find $\sqrt[3]{\ }$ on calc)

$\boxed{\text{Altitude} = 3.58\times10^4 \text{ Km} = 35800 \text{ Km}}$

b) What is the orbital velocity of a satellite that has a period of 24 hours.

$$V_{ORBIT} = \sqrt{\frac{Gm}{R}} = \sqrt{\frac{(6.67\times10^{-11})(5.98\times10^{24})}{4.22\times10^7}} = 3074 \text{ m/s}.$$

c) What do we call this type of orbit?

Geosynchronous orbit.