Physics U6 Circular Motion Worksheet #4

Potential Energy

Name: ______________________

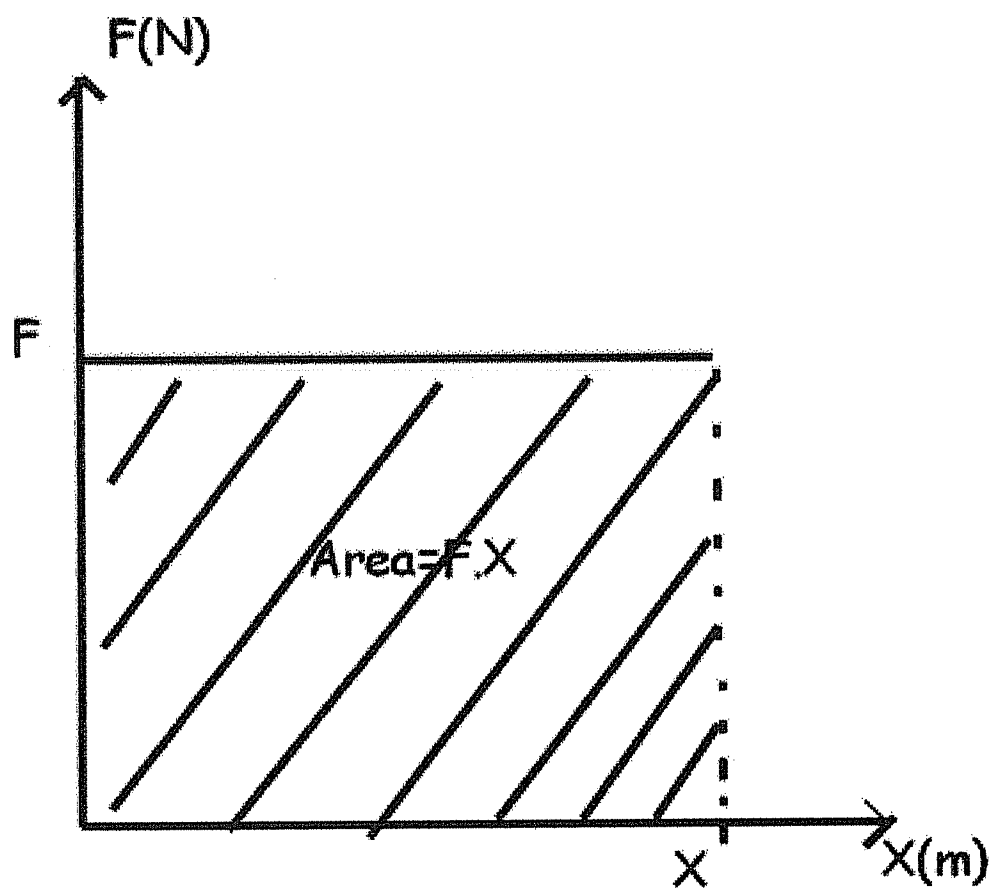
In Physics 11 we always used PE=Ep=mgh

- this is the gravitational potential energy of a mass at a certain height

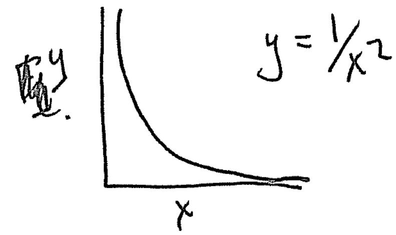
If we lift a box from the floor to the table top, the force of gravity is constant.

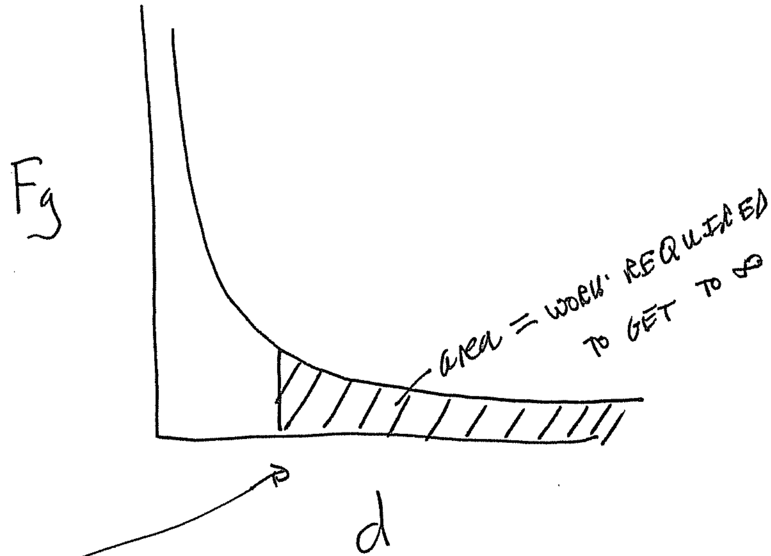
If we lift a box from the surface of the Earth to 50, 000 km out, the force of gravity is not constant, Fg gets less. We need a way to deal with work done when the force is changing.

We know that the work done is equal to the area under a Force vs distance graph = work done or energy.



If we look at the Force of gravity vs distance (you made a graph with this shape previously) we see that it is curved.





To find this area we need to be able to deal with a changing force, during labs you have added up the area of graphs similar to this by constructing multiple rectangles. Calculus give us this nice clean formula for the area under the curve . .

$$Ep = -GMM/R$$

The negative sign is there because instead of taking the ground at PE = 0, we take PE=0 at infinity, a more universal reference point for space problems in general.

1. Calculate the potential energy for a 600 kg mass on the surface of the Earth.

$$E_p = \frac{-GMm}{R} = \frac{-(6.67\times10^{-11})(600)(5.98\times10^{24})}{6.38\times10^{6}}$$

$$= -3.751\times10^{10} \text{ Joules}$$

2. Calculate the potential energy for a 600 kg mass at an altitude of 1200 km above the Earth.

$$R_{TOT} = 6.38\times10^{6} + 1{,}200{,}000\,m = 7.58\times10^{6}$$

$$E_p = -\frac{GMm}{R} = \frac{}{7.58\times10^{6}} = -3.157\times10^{10} \text{ Joules}$$

3. Calculate the work required to lift a 600 kg mass from the surface of the Earth to an altitude of 1200 km.

$$\text{WORK} = \Delta E_p = E_{p_f} - E_{p_i} = (-3.157\times10^{10}) - (-3.751\times10^{10})$$

$$= 5.94\times10^{9} \text{ Joules}.$$

initial vel=0 so $KE_i = 0$

4. Calculate the impact speed for a 550 kg object "dropped" from an altitude of 900 km. Assume not energy is converted to heat.

$R_i = 6.38 \times 10^6 + 500{,}000\,m = 6.88 \times 10^6\,m$

$$E_i = E_f$$

$$\cancel{KE_i}^{\,=0} + PE_i = KE_f + PE_f$$

$$\frac{-(6.67 \times 10^{-11})(550)(5.98 \times 10^{24})}{6.88 \times 10^6} = KE_f + \frac{-(6.67 \times 10^{-11})(550)(5.98 \times 10^{24})}{6.38 \times 10^6}$$

$$-3.1886 \times 10^{10} = KE_f + -3.4385 \times 10^{10}$$

$$KE_f = 2.499 \times 10^9 = \tfrac{1}{2} m v^2 = \tfrac{1}{2}(550)(V)^2$$

$$Vel = 3.01 \times 10^3 = 3010\ m/s$$

5. Calculate the impact speed of for a 7300 kg rocket that falls to Earth from a altitude of 350 km, assume that during the fall that 1.9 x 10^{10} Joules of energy are converted to heat.

$R_{TOT} = R_E + 350{,}000\,m = 6.73 \times 10^6$

$$E_i = E_f$$

$$PE_i = PE_f + KE_f + Heat$$

$$\frac{-(6.67 \times 10^{-11})(7300)(5.98 \times 10^{24})}{6.73 \times 10^6} = \frac{-(6.67 \times 10^{-11})(7300)(5.98 \times 10^{24})}{6.38 \times 10^6} + KE_f + 1.9 \times 10^{10}$$

$$-4.3265 \times 10^{11} = -4.5842 \times 10^{11} + KE_f + 1.9 \times 10^{10}$$

$$KE_f = 6.77 \times 10^9 \text{ Joules} = \tfrac{1}{2} m v^2 = \tfrac{1}{2}(7300)(vel)^2$$

$$vel = 1.36 \times 10^3 = 1360\,m/s.$$

3.1×10^7 m

6. A 2000 kg satellite is in a circular orbit around the Earth. The satellite has a speed of 3600 m/s orbiting at a radius of 3.1 x 10⁴km. What is the total energy of the satellite (KE+PE)?

$$E_{TOT} = KE + PE = \frac{1}{2}mv^2 + -\frac{Gmm_1}{R}$$

$$= \frac{1}{2}(2000)(3600)^2 + \frac{(-6.67\times10^{-11})(2000)(5.98\times10^{24})}{3.1\times10^7}$$

$$= 1.296 + 10^{10} - 2.573\times10^{10}$$

$$= -1.28\times10^{10} \text{ Joules}$$

In general,

$$E_{TOT} = \frac{1}{2}mv^2 - \frac{Gmm}{R}$$

but orbital $vel^2 = \frac{Gm}{R}$

so

$$E_{TOT} = \frac{1}{2}m\left(\frac{Gm}{R}\right) - \frac{Gmm}{R}$$

$$= \frac{1}{2}\frac{Gmm}{R} - \frac{Gmm}{R} = -\frac{1}{2}\frac{Gmm}{R}$$

OR $= -\frac{1}{2}E_p + E_p = \frac{1}{2}E_p$

7. Find work required to lift the vehicle.

What minimum energy is required to raise a 1.7×10^3 kg vehicle from the surface of the Moon to a height of 5.22×10^6 m?

A. 1.6×10^9 J
B. 3.6×10^9 J
C. 4.8×10^9 J
D. 1.4×10^{10} J

R_f: $5.22\times10^6 + 1.74\times10^6 = 6.96\times10^6$

$$\text{WORK} = \Delta E_p = E_{p_f} - E_{p_i}$$

$$= \frac{-(6.67\times10^{-11})(1.7\times10^3)(7.35\times10^{22})}{6.96\times10^6} - \frac{\text{" "}}{1.74\times10^6}$$

$$= -1.1974\times10^9 + 4.7898\times10^9$$

$$= 3.59\times10^9$$

8. Change in energy = ?

What minimum energy is required to take a stationary 3.5×10^{3} kg satellite from the surface of the Earth and put it into a circular orbit with a radius of 6.88×10^{6} m and an orbital speed of 7.61×10^{3} m/s? (Ignore Earth's rotation.) **(7 marks)**

$$\text{Work} = E_f - E_i = KE_f + PE_f - PE_i$$

$$= \tfrac{1}{2}(3.5\times10^{3})(7.61\times10^{3})^{2} + \frac{(-6.67\times10^{-11})(3.5\times10^{3})(5.98\times10^{24})}{6.88\times10^{6}} - \frac{(-6.67\times10^{-11})(3500)(5.98\times10^{24})}{6.38\times10^{6}}$$

$$= 1.0135\times10^{11} - 2.0291\times10^{11} + 2.1881\times10^{11}$$

$$= 1.17\times10^{11} \text{ Joules.}$$